Mixedness and Entanglement in the presence of Localized Closed Timelike Curves

Eylee Jung[1] and DaeKil Park[1,2]

[1] *Department of Electronic Engineering, Kyungnam University, Changwon, 631-701, Korea*

[2] *Department of Physics, Kyungnam University, Changwon, 631-701, Korea*

Abstract

We examine mixedness and entanglement of the chronology-respecting (CR) system with assuming that quantum mechanical closed timelike curves (CTCs) exist in nature and by introducing the qubit system and applying the general controlled operations between CR and CTC systems. We use the magnitude of Bloch vector as a measure of mixedness. While Deutschian-CTC (D-CTC) either preserves or decreases the magnitude, postselected-CTC (P-CTC) can increases it. Nonintuitively, even the completely mixed CR-qubit can be converted into a pure state after CTC-qubit travels around the P-CTC. It is also shown that while D-CTC cannot increase the entanglement of CR system, P-CTC can increase it. Surprisingly, any partially entangled state can be maximally entangled pure state if P-CTC exists. Thus, distillation of P-CTC-assisted entanglement can be easily achieved without preparing the multiple copies of the partially entangled state.

Introduction.—It is well-known that the theory of general relativity allows the possible existence of closed timelike curves (CTCs)[1–3]. However, allowance of time travel generates logical paradoxes such as the *grandfather paradox*. Deutsch[4] solved this paradox from an aspect of quantum information theories by deriving self-consistent equation of CTC interaction. Thus, it makes it possible to explore the properties of CTCs without relying on the exotic spacetime geometries.

Then, it is natural to ask how quantum mechanics is modified if Deutsch's CTCs (D-CTCs) exist. For last few years this question was explored in the various contexts[5–11]. Among them most striking result is that any non-orthogonal states can be perfectly distinguished if one can access to D-CTCs[8]. This fact implies that Security of usual quantum cryptography scheme such as BB84 protocol[12] is not guaranteed. Subsequently, the authors of Ref.[13] raised a question on the perfect discrimination and computational power in the presence of D-CTCs. They argued that when the input state is a labeled mixture, the assistance of CTCs in distinguishability and computational power is of no use. However, their argument was also criticized in Ref.[14]. The authors of Ref.[14] claimed by constructing the equivalent circuit that the CTCs would be a true powerful resource for quantum information processing. Another nonintuitive result arising due to existence of D-CTCs is that any arbitrary dimensional quantum states can be perfectly cloned if the dimension of the CTC system is infinite[10, 11]. Thus, the well-known no-cloning theorem[15] can be broken in the presence of D-CTCs.

The Deutsch's self-consistency condition is expressed as

$$\rho_{out}^{(CTC)} \equiv \mathrm{tr}_{CR}\left[U\left(\rho_{in}^{(CR)} \otimes \rho_{in}^{(CTC)}\right)U^{\dagger}\right] = \rho_{in}^{(CTC)} \tag{1}$$

where $\rho_{in}^{(CR)}$ and $\rho_{in}^{(CTC)}$ are input states of the chronology-respecting (CR) and chronology-violating systems, respectively. The operator U represents the unitary interaction between CR and CTC systems. Deutsch showed[4] that the fixed-point solution of Eq. (1) always exists, but it does not necessarily have to be unique. If there are many solutions, Deutsch suggested the *maximum entropy rule*. If $\rho^{(CTC)}$ is fixed, the CR system is evolved as

$$\rho_{in}^{(CR)} \to \rho_{out}^{(CR)} \equiv \mathrm{tr}_{CTC}\left[U\left(\rho_{in}^{(CR)} \otimes \rho_{in}^{(CTC)}\right)U^{\dagger}\right]. \tag{2}$$

The output state $\rho_{out}^{(CR)}$ is in general non-unitary evolution of $\rho_{in}^{(CR)}$, because $\rho_{out}^{(CR)}$ depends on both $\rho_{in}^{(CR)}$ and $\rho^{(CTC)}$, and $\rho^{(CTC)}$ also depends on $\rho_{in}^{(CR)}$.

The post-selected CTCs[16–18] (P-CTCs) are another type of quantum mechanical CTCs, which also solve the paradoxes. P-CTCs provide a self-consistent picture of quantum mechanical time travel via post-selected quantum teleportation[19]. It is based on the Horowitz-Maldacena "final state condition"[20] for black hole evaporation[21] and are consistent with path-integral approaches to CTCs[22, 23]. In P-CTCs formalism the state in CTC-system is not explicitly specified while the output state of the CR-system is given by

$$\rho_{out}^{(CR)} \propto V \rho_{in}^{(CR)} V^{\dagger} \tag{3}$$

where $V = \mathrm{tr}_{CTC} U$. It turned out that though P-CTCs are less powerful resource than D-CTCs in the quantum information processing, they also have a computational and discrimination power[24].

In this Letter we explore the following issues. By introducing simple qubit system and general controlled operations mixedness of the CR system is examined. The mixedness is measured by a magnitude of Bloch vector. It is shown that the magnitude of Bloch vector for qubit system assisted by D-CTCs either preserves or decreases. Thus, the pure CR-state can propagate to mixed state when CTC-qubit travels around the D-CTC. In this sense CTC-problem resembles the information loss problem[25, 26] in Hawking radiation. For P-CTCs, however, the magnitude of Bloch vector can increase. In this case a mixed state can evolve to a pure state. Even the completely mixed state can be converted into a pure state if the controlled operation is chosen appropriately. We also examine how the entanglement of the CR-system is changed in the presence of CTCs. While D-CTCs always either preserve or degrade the entanglement, P-CTCs can increase it. Surprisingly, if any partially entangled CR-state is prepared, one can change it into a maximally entangled pure state if P-CTCs assist. This fact implies that distillation of entanglement[27, 28] can be easily achieved without preparing the many copies of the partially entangled state if P-CTCs are appropriately exploited.

Mixedness.—We examine the mixedness of the CR-system when there is an interaction of the controlled-U_2 operation between CR and CTC systems, where U_2 is represented by four real parameters as follows;

$$U_2 = e^{i\phi/2} \begin{pmatrix} \cos\theta e^{i\phi_1} & \sin\theta e^{i\phi_2} \\ -\sin\theta e^{-i\phi_2} & \cos\theta e^{-i\phi_1} \end{pmatrix}. \tag{4}$$

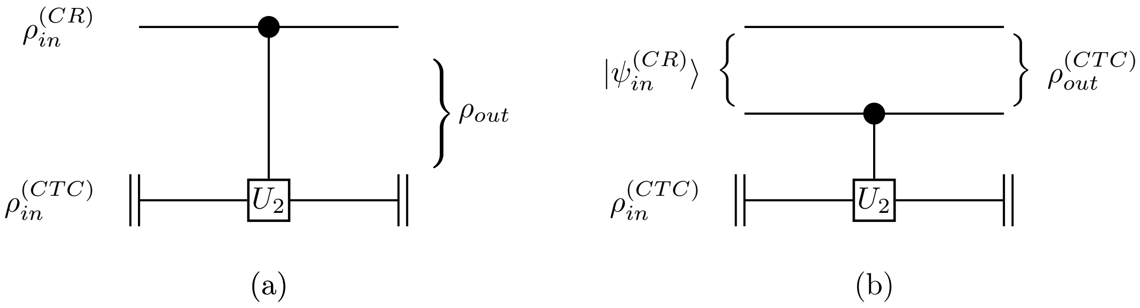

FIG. 1: (a) Circuit for examining the mixedness of CR-system when CR and CTC systems interact with each other through general controlled operations. The U_2 is represented by Eq. (4). The double vertical bars on the bottom left and right indicate the past and future mouths of the wormhole for the CTC. (b) Circuit for examining the entanglement of CR-state in the presence of CTC. We choose the initial CR-state as a partially entangled state $|\psi_{in}^{(CR)}\rangle = \alpha|00\rangle + \beta|11\rangle$ with $\alpha^2 + \beta^2 = 1$ and $|\beta| \geq |\alpha|$.

The initial CR-state is chosen as a general form of one-qubit $\rho_{in}^{(CR)} = \frac{1}{2}(I_2 + \boldsymbol{r} \cdot \boldsymbol{\sigma})$, where $|\boldsymbol{r}| = 0$ and $|\boldsymbol{r}| = 1$ correspond to the completely mixed and pure states, respectively. We assume $r_3 \neq 1$ because if $r_3 = 1$, the controlled operation cannot be turned on.

For the case of P-CTC one can derive the output-CR state by making use of Eq. (3) as $\rho_{out}^{CR} = \frac{1}{2}(I_2 + \boldsymbol{r'} \cdot \boldsymbol{\sigma})$, where

$$\begin{aligned} r'_1 &= \frac{2\cos\theta\cos\phi_1}{(1+r_3) + \cos^2\theta\cos^2\phi_1(1-r_3)} \left(r_1 \cos\frac{\phi}{2} - r_2 \sin\frac{\phi}{2} \right) \\ r'_2 &= \frac{2\cos\theta\cos\phi_1}{(1+r_3) + \cos^2\theta\cos^2\phi_1(1-r_3)} \left(r_1 \sin\frac{\phi}{2} + r_2 \cos\frac{\phi}{2} \right) \\ r'_3 &= \frac{(1+r_3) - \cos^2\theta\cos^2\phi_1(1-r_3)}{(1+r_3) + \cos^2\theta\cos^2\phi_1(1-r_3)}. \end{aligned} \tag{5}$$

Then, one can show directly

$$|\boldsymbol{r'}|^2 - |\boldsymbol{r}|^2 = (1 - |\boldsymbol{r}|^2) \left[1 - \left(\frac{2\cos\theta\cos\phi_1}{(1+r_3) + \cos^2\theta\cos^2\phi_1(1-r_3)} \right)^2 \right]. \tag{6}$$

As expected, Eq. (6) guarantees that the pure input CR-state always evolves into pure. Since, however, the right-hand side of Eq. (6) can be positive or negative depending on U_2, the CR-state can evolve with increasing or decreasing its mixedness. Even though $\rho_{in}^{(CR)}$ is completely mixed state, $\rho_{out}^{(CR)}$ becomes pure state $|0\rangle\langle 0|$ when $\theta = \pi/2$ or $\phi_1 = \pi/2$. Thus, P-CTC allows the evolution from mixed to pure state if qubit travels around the P-CTC.

However, the situation is different if the CR-system is assisted by D-CTC. If the initial CTC-state $\rho_{in}^{(CTC)}$ is chosen as a general form $\rho_{in}^{(CTC)} = \frac{1}{2}(I_2 + \boldsymbol{s} \cdot \boldsymbol{\sigma})$, one can show directly $\rho_{out}^{(CTC)} \equiv \text{tr}_{CR}\left[U\rho_{in}^{(CR)} \otimes \rho_{in}^{(CTC)} U^\dagger\right] = \frac{1}{2}(I_2 + \boldsymbol{s'} \cdot \boldsymbol{\sigma})$, where

$$\begin{aligned}
\Delta s_1 &= -(1 - r_3)\Big[s_1\left(\sin^2\phi_1 + \sin^2\theta\cos(\phi_1+\phi_2)\cos(\phi_1-\phi_2)\right) \\
&-s_2\left(\cos^2\theta\sin\phi_1\cos\phi_1 + \sin^2\theta\sin\phi_2\cos\phi_2\right) + s_3\sin\theta\cos\theta\cos(\phi_1+\phi_2)\Big] \\
\Delta s_2 &= -(1 - r_3)\Big[s_1\left(\cos^2\theta\sin\phi_1\cos\phi_1 - \sin^2\theta\sin\phi_2\cos\phi_2\right) \\
&+s_2\left(\sin^2\phi_1 - \sin^2\theta\sin(\phi_1+\phi_2)\sin(\phi_1-\phi_2)\right) - s_3\sin\theta\cos\theta\sin(\phi_1+\phi_2)\Big] \\
\Delta s_3 &= (1 - r_3)\sin\theta\Big[s_1\cos\theta\cos(\phi_1-\phi_2) + s_2\cos\theta\sin(\phi_1-\phi_2) - s_3\sin\theta\Big]
\end{aligned} \tag{7}$$

with $\Delta s_j = s'_j - s_j \ (j = 1, 2, 3)$. Then, the self-consistency condition (1) simply reduces to $\Delta s_j = 0$.

condition	solution of self-consistency condition
$\sin\theta = 0, \sin\phi_1 = 0$	no constraint
$\sin\theta = 0, \sin\phi_1 \neq 0$	$s_1 = s_2 = 0$
$\sin\theta \neq 0, \sin\phi_1 = 0$	$s_1 = s_2\tan\phi_2, s_3 = 0$
$\sin\theta \neq 0, \sin\phi_1 \neq 0$	$s_1 = s_3\tan\theta\csc\phi_1\sin\phi_2, s_2 = s_3\tan\theta\csc\phi_1\cos\phi_2$

Table I: Solutions of the self-consistency condition for various U_2.

The solutions of the self-consistency condition is summarized in Table I for various U_2. In this Letter we discuss only the case of $\sin\theta \neq 0$ and $\sin\phi_1 \neq 0$, because the remaining cases can be discussed similarly. Since $|\boldsymbol{s}| \leq 1$, the self-consistency condition implies

$$s_3^2 \leq \frac{\sin^2\phi_1}{\sin^2\phi_1 + \tan^2\theta} \tag{8}$$

where equality holds for pure CTC-state. Then, the output CR-state becomes $\rho_{out}^{(CR)} \equiv \text{tr}_{CTC}\left[U\rho_{in}^{(CR)} \otimes \rho_{in}^{(CTC)} U^\dagger\right] = \frac{1}{2}(I_2 + \boldsymbol{r'} \cdot \boldsymbol{\sigma})$, where

$$r'_1 = Pr_1 - Qr_2 \qquad r'_2 = Qr_1 + Pr_2 \qquad r'_3 = r_3 \tag{9}$$

with

$$\begin{aligned}
P &= \cos\frac{\phi}{2}\cos\theta\cos\phi_1 - s_3\sin\frac{\phi}{2}\frac{\sin^2\theta + \cos^2\theta\sin^2\phi_1}{\cos\theta\sin\phi_1} \\
Q &= \sin\frac{\phi}{2}\cos\theta\cos\phi_1 + s_3\cos\frac{\phi}{2}\frac{\sin^2\theta + \cos^2\theta\sin^2\phi_1}{\cos\theta\sin\phi_1}.
\end{aligned} \tag{10}$$

Therefore, $|\boldsymbol{r}'|^2 = (P^2 + Q^2)(r_1^2 + r_2^2) + r_3^2$, where

$$P^2 + Q^2 = \cos^2\theta\cos^2\phi_1 + s_3^2\left(\frac{\sin^2\theta + \cos^2\theta\sin^2\phi_1)}{\cos\theta\sin\phi_1}\right)^2.$$

When s_3^2 saturates the inequality (8), it is easy to show $|\boldsymbol{r}'| = |\boldsymbol{r}|$. Thus, the mixedness of the CR-system is preserved when the CTC-system is pure. When CTC-state is mixed, $\rho_{out}^{(CR)}$ is more mixed than $\rho_{in}^{(CR)}$, i.e. $|\boldsymbol{r}'| < |\boldsymbol{r}|$. If the Deutsch's maximum entropy postulate is chosen, $\rho_{out}^{(CR)}$ becomes the maximal mixed state $|\boldsymbol{r}'|^2 = \cos^2\theta\cos^2\phi_1(r_1^2 + r_2^2) + r_3^2$. Thus, any pure states of the form $\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\theta}|1\rangle\right)$ can be converted into the completely mixed state when $\cos\theta = 0$ or $\cos\phi_1 = 0$ if maximum entropy rule is chosen.

Entanglement—We examine how the entanglement of CR-system is changed in the presence of CTCs. To explore this issue we introduce partially entangled two-qubit initial state $|\psi_{in}^{(CR)}\rangle = \alpha|00\rangle + \beta|11\rangle$ where $\alpha^2 + \beta^2 = 1$. We also choose $|\beta| \geq |\alpha|$ without loss of generality. One party of CR-system interacts with CTC through the controlled-U_2 operation. The other party has no interaction with the CTC-system. This situation is depicted in Fig. 1(b) as a quantum circuit. We will use the concurrence[29] as an entanglement measure. The concurrence of $|\psi_{in}^{(CR)}\rangle$ is $2|\alpha\beta|$.

For the case of P-CTC one can derive $\rho_{out}^{(CR)}$ by making use of Eq. (3) in a form

$$\rho_{out}^{(CR)} = \frac{1}{\alpha^2 + \beta^2\cos^2\theta\cos^2\phi_1}\Big[\alpha^2|00\rangle\langle 00| + \beta^2\cos^2\theta\cos^2\phi_1|11\rangle\langle 11| \\ + \alpha\beta e^{-i\phi/2}\cos\theta\cos\phi_1|00\rangle\langle 11| + \alpha\beta e^{i\phi/2}\cos\theta\cos\phi_1|11\rangle\langle 00|\Big]. \quad (11)$$

The concurrence of $\rho_{out}^{(CR)}$ is easily computed by following the procedure of Ref.[29] and final expression is

$$\mathcal{C}\left(\rho_{out}^{(CR)}\right) = 2|\alpha\beta|\gamma \quad (12)$$

where the ratio γ is

$$\gamma = \frac{|\cos\theta\cos\phi_1|}{\alpha^2 + \beta^2\cos^2\theta\cos^2\phi_1}. \quad (13)$$

It is remarkable to note that the ratio γ is dependent on both U_2 and the initial CR-state. Surprisingly, one can always make $\rho_{out}^{(CR)}$ maximally entangled pure state $\frac{1}{\sqrt{2}}(|00\rangle \pm e^{i\phi/2}|11\rangle)$ by choosing $\cos\theta\cos\phi_1 = \pm\frac{\alpha}{\beta}$. Thus, if P-CTC exists, the distillation of entanglement of CR-system can be easily achieved without preparing multiple copies of the partially entangled state. It is sufficient to prepare a single copy for complete distillation by choosing U_2 appropriately.

The situation is different for the case of D-CTC. We define the initial CTC-state as a one-qubit general form $\rho_{in}^{(CTC)} = \frac{1}{2}(I_2 + \boldsymbol{s} \cdot \boldsymbol{\sigma})$. Then, the output CTC state becomes $\rho_{out}^{(CTC)} \equiv \text{tr}_{CR}\left[U|\psi_{in}^{(CR)}\rangle\langle\psi_{in}^{(CR)}|\otimes\rho_{in}^{(CTC)}U^\dagger\right] = \frac{1}{2}(I_2 + \boldsymbol{s}' \cdot \boldsymbol{\sigma})$, where Δs_j $(j = 1, 2, 3)$ are exactly the same with Eq. (7) if $1 - r_3$ is changed into $2\beta^2$. Thus, the solutions of the self-consistency condition are identical with those given in Table I. One can also show directly that the output CR-state is

$$\rho_{out}^{(CR)} \equiv \text{tr}_{CTC}\left[U|\psi_{in}^{(CR)}\rangle\langle\psi_{in}^{(CR)}|\otimes\rho_{in}^{(CTC)}U^\dagger\right] = \alpha^2|00\rangle\langle 00| + \beta^2|11\rangle\langle 11| + A|00\rangle\langle 11| + A^*|11\rangle\langle 00| \tag{14}$$

where

$$A = e^{-i\phi/2}\alpha\beta\left[\cos\theta\cos\phi_1 - i\left(s_1\sin\theta\sin\phi_2 + s_2\sin\theta\cos\phi_2 + s_3\cos\theta\sin\phi_1\right)\right]. \tag{15}$$

It is easy to show that the concurrence of $\rho_{out}^{(CR)}$ is

$$\mathcal{C}\left(\rho_{out}^{(CR)}\right) = 2\min\left(|A|, |\alpha\beta|\right). \tag{16}$$

Thus, D-CTC can either preserve or decrease the entanglement of CR-system.

For example, let us consider the case of $\sin\theta \neq 0$ and $\sin\phi_1 \neq 0$. Then, the variation of entanglement $\Delta\mathcal{E} \equiv \mathcal{C}\left(\rho_{in}^{(CR)}\right) - \mathcal{C}\left(\rho_{out}^{(CR)}\right)$ can be computed by making use of Eq. (16) and Table I:

$$\Delta\mathcal{E} = 2|\alpha\beta|\left[1 - \sqrt{1 - \left(1 - \frac{\sin^2\phi_1 + \tan^2\theta}{\sin^2\phi_1}s_3^2\right)\left(\sin^2\theta + \cos^2\theta\sin^2\phi_1\right)}\right]. \tag{17}$$

Thus, if the inequality (8) is saturated, $\Delta\mathcal{E}$ vanishes. This means that if the CTC-state is pure, the entanglement of CR-state is preserved. If we choose the maximal entropy CTC-state as Deutsch suggested, the maximal degradation of entanglement $\Delta\mathcal{E} = 2|\alpha\beta|(1 - |\cos\theta\cos\phi_1|)$ occurs.

Conclusions—Although the theory of general relativity does allow CTC as a solution of Einstein field equation, still there are a lot of controversial for existence of CTCs. In this Letter we have addressed two issues, mixedness and entanglement for CR system with assuming that D-CTC and/or P-CTC exist(s) in nature. It was shown that while D-CTC-assisted qubit cannot increase the magnitude of its Bloch vector, P-CTC-assisted qubit can. As a result, the mixed CR-state can evolve to pure CR-state if P-CTC exists. Even the completely mixed state can evolve to pure state if we choose the phase angles of U_2 appropriately.

Although the CTC-state is not specified explicitly for the case of P-CTC, one can get some information of P-CTC-state if exists any. Let us imagine a closed system consists of CR and P-CTC subsystems. Let us assume that they interacts with each other through some unitary operation. If one uses the subadditivity of the von Neumann entropy one can show $\Delta S^{(CTC)} \geq -\Delta S^{(CR)}$, where S is a von Neumann entropy and $\Delta S^{(\cdot)} \equiv S\left(\rho_{out}^{(\cdot)}\right) - S\left(\rho_{in}^{(\cdot)}\right)$. Thus, computing the entropy difference of CR-subsystem one can compute the lower bound of $\Delta S^{(CTC)}$ although we do not know the P-CTC state explicitly.

We also have studied the case where the CR-system consists of bipartite partially entangled particles and one of them interacts with CTC system through controlled-U_2 operation. For the case of P-CTC surprisingly the partially entangled state can always be converted into the maximally entangled pure state if the phase angles of U_2 are chosen appropriately. If, therefore, P-CTCs exist, the distillation protocol of entanglement is easily achieved without preparing the multiple copies of the partially entangled state. For the case of D-CTC such a nonintuitive effect disappears because D-CTC either preserves or decreases the entanglement of CR system.

There are a lot of questions in the context of CTCs. How to incorporate the general relativistic CTCs into the quantum mechanical CTCs or *vice versa*? What happens to the uncertainty relations if CTCs exist? Does black hole physics be modified if CTCs exist? Are the thermodynamic laws still valid even if CTCs exist? Probably, the theory of quantum gravity may give some answers in the future.

Acknowledgments

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2011-0011971).

[1] K. Ǵodel, *An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation*, Rev. Mod. Phys. **21**, 447 (1949).

[2] M. S. Morris, K. S. Thorne, and U. Yurtsever, *Wormholes, Time Machines, and the Weak Energy Condition*, Phys. Rev. Lett. **61**, 1446 (1988).

[3] J. R. Gott, *Closed timelike curves produced by pairs of moving cosmic strings: Exact solutions*, Phys. Rev. Lett. **66**, 1126 (1991).

[4] D. Deutsch, *Quantum mechanics near closed timelike lines*, Phys. Rev. **D 44**, 3197 (1991).

[5] T. A. Brun, *Computers with Closed timelike curves can solve hard problems efficiently*, Found. Phys. Lett. **16**, 345 (2003).

[6] D. Bacon, *Quantum computational complexity in the presence of closed timelike curves*, Phys. Rev. **A 70**, 032309 (2004).

[7] S. Aaronson and J. Watrous, *Closed timelike curves make quantum and classical computing equivalent*, Proc. R. Soc. **A 465** 631 (2009).

[8] T. A. Brun, J. Harrington, and M. M. Wilde, *Localized Closed Timelike Curves Can Perfectly Distinguish Quantum States*, Phys. Rev. Lett. **102**, 210402 (2009).

[9] A. K. Pati, I. Chakrabarty, and P. Agrawal, *Purification of mixed states with closed timelike curve is not possible*, Phys. Rev. **A 84**, 062325 (2011).

[10] D. Ahn, C. R. Myers, T. C. Ralph, and R. B. Mann, *Quantum-state cloning in the presence of a closed timelike curve*, Phys. Rev. **A 88**, 022332 (2013).

[11] T. A. Brun, M. M. Wilde, and A. Winter, *Quantum State Cloning Using Deutschian Closed Timelike Curves*, Phys. Rev. Lett. **111**, 190401 (2013).

[12] C. H. Bennett and G. Brassard, *Quantum Cryptography, Public Key Distribution and Coin Tossings*, in Proceedings of the IEEE International Conference on Computer, Systems, and Signal Processing, Bangalore, India (IEEE, New York, 1984), pp. 175-179.

[13] C. H. Bennett, D. Leung, G. Smith, and J. A. Smolin, *Can Closed Timelike Curves or Nonlinear Quantum Mechanics Improve Quantum State Discrimination or Help Solve Hard Problems?*, Phys. Rev. Lett. **103**, 170502 (2009).

[14] T. C. Ralph and C. R. Myers, *Information flow of quantum states interacting with closed timelike curves*, Phys. Rev. **A 82**, 062330 (2010).

[15] W. K. Wootters and W. H. Zurek, *A single quantum cannot be cloned*, Nature, **299**, 802 (1982).

[16] G. Svetlichny, *Time Travel: Deutsch vs. Teleportation*, Int. J. Theor. Phys. **50**, 3903 (2011).

[17] S. Lloyd *et al*, *Closed Timelike Curves vis Postselection: Theory and Experimental Test of Consistency*, Phys. Rev. Lett. **106**, 040403 (2011).

[18] S. Lloyd *et al*, *Quantum mechanics of time travel through post-selected teleportation*, Phys.

Rev. **D 84**, 025007 (2011).

[19] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, *Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channles*, Phys. Rev. Lett. **70** (1993) 1895.

[20] G. T. Horowitz and J. Maldacena, *The black hole final state*, JHEP **0402**, 008 (2004).

[21] S. W. Hawking, *Black hole explosions?*, Nature bf 248, 30 (1974).

[22] H. D. Politzer, *Simple quantum systems in spacetimes with closed timelike curves*, Phys. Rev. **D 46**, 4470 (1992).

[23] H. D. Politzer, *Path integrals, density matrices, and information flow with closed timelike curves*, Phys. Rev. **D 49**, 3981 (1994).

[24] T. A. Brun and M. M. Wilde, *Perfect State Distinguishability and Computational Speedups with Postselected Closed Timelike Curves*, Found. Phys. **42**, 341 (2012).

[25] S. W. Hawking, *Breakdown of predictability in gravitational collapse*, Phys. Rev. **D 14**, 2460 (1976).

[26] S. W. Hawking, *Information loss in black holes*, Phys. Rev. **D 72**, 084013 (2005).

[27] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, *Purification of Noisy Entanglement and Faithful Teleportation via Noisy Channels*, Phys. Rev. Lett. **76**, 722 (1996).

[28] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, *Mixed-state entanglement and quantum error correction*, Phys. Rev. bf A 54, 3824 (1996).

[29] W. K. Wootters, *Entanglement of Formation of an Arbitrary State of Two Qubits*, Phys. Rev. Lett. **80**, 2245 (1998).